Math 270: Differential Equations Day 10 Part 2

Section 4.4: The Method of Undetermined Coefficients (Part 1)

Section 4.4: The Method of Undetermined Coefficients (Part 1)

Now we turn our attention to solving **non-homogeneous** 2nd-order linear DEs with constant coefficients $ay'' + by' + cy = f(t)$

Results:

Let y_p be one particular solution to the non-homogeneous DE $ay'' + by' + cy = f(t)$

1) If y_h is any solution to the homogeneous DE $ay'' + by' + cy = 0$,
   then $y = y_h + y_p$ is a solution to the non-homogeneous DE $ay'' + by' + cy = f(t)$

Prove…

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2) If y is any solution to the non-homogeneous DE $ay'' + by' + cy = f(t)$,
   then $y - y_p$ is a solution to the homogeneous DE $ay'' + by' + cy = 0$

Prove…

Section 4.4: The Method of Undetermined Coefficients (Part 1)

To solve 2nd-order linear non-homogeneous DEs with constant coefficients:

1) Guess **a** particular solution y_p to the non-homogeneous DE $ay'' + by' + cy = f(t)$
2) Find **all** solutions y_h to the homogeneous DE $ay'' + by' + cy = 0$
3) Then **all** solutions to the non-homogeneous DE are $y = y_h + y_p$

How do we guess y_p?

Section 4.4: The Method of Undetermined Coefficients (Part 1)

Example 1 Find a particular solution to $y'' + 3y' + 2y = 3t$.

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Example 2 Find a particular solution to $y'' + 3y' + 2y = 10e^{3t}$.

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Example 3 Find a particular solution to $y'' + 3y' + 2y = \sin t$.

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Example 4 Find a particular solution to $y'' + 4y = 5t^2e^t$.